THE ONSET OF NUCLEATE BOILING IN FORCED LIQUID FLOW

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We have derived the functions to calculate the difference between the wall temperature and the temperature of liquid saturation, corresponding to the onset of nucleate boiling, and we have determined the areas of applicability for these functions.

A characteristic feature of gas-separation heat exchangers operating in a liquid vaporization regime is the small difference in temperatures between the heating surface and the vaporized liquid. Consequently, a particularly urgent problem is the determination of the minimum temperature difference at which nucleate boiling sets in, in particular, under conditions of forced convection in tubes.

According to the kinetic theory of liquids [1], the critical dimension for the formation of a new phase is governed by the parameters of state for the liquid on the saturation line, in addition to its properties and the degree of superheating.

The condition of mechanical equilibrium for the nucleus of the new phase (spherical in shape) is given by the expression

$$P_a - P_s = \frac{2\sigma}{R}. \tag{1}$$

Using the relationship between temperature and pressure at the saturation line (the Clapeyron–Clausius equation)

$$\frac{dP_s}{dT_s} = \frac{r}{T_s(v'' - v')}, \tag{2}$$

in the case of low liquid superheating, the finite pressure difference can be replaced in (1) by the finite temperature difference.

In accordance with (2) we find

$$\frac{r(T_a - T_s)}{T_s(v'' - v')} = \frac{2\sigma}{R}, \tag{3}$$

from which the critical radius of formation for the new phase, in the case of low values for $(T_a - T_s)$, can be expressed as

$$R_{cr} = \frac{2\sigma T_s(v'' - v')}{r(T_a - T_s)}. \tag{4}$$

We assume that the critical formation of the new phase in the region in which there is a temperature gradient in a direction normal to the heating surface is capable of continuing its growth if the temperature of the superheated liquid is given by T_a when $y_a = 2R_{cr}$. Otherwise, the bubble will not grow. In the formation of bubbles on a solid surface, instead of $y_a = 2R_{cr}$ reference [2] recommends the introduction of

$$y_a = R_{cr}(1 + \cos\Theta) = CR_{cr}. \tag{5}$$

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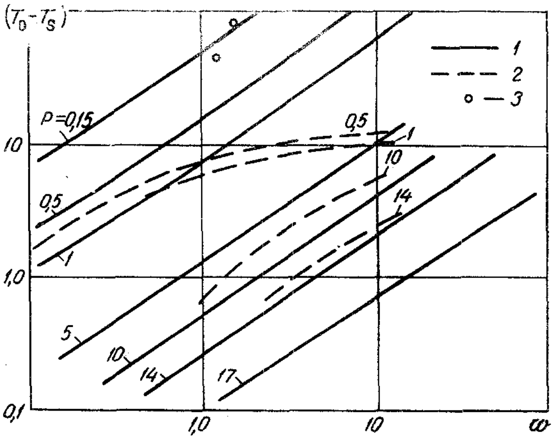

Fig. 1. Boundary for the temperature (deg) of wall superheating, leading to the onset of nucleate boiling, as a function of flow velocity (w, m/sec; P, MN/m^2): 1) from the theoretical formula (24); 2) from the relationship for the onset of boiling in a subcooled liquid [6]; 3) experimental data from [7].

To some extent, this makes provision for the change in the height of formation as compared to a complete sphere, because of the proximity of the heating surface. Having introduced (5) into (4), we find from the latter that

$$T_a = T_s + \frac{2\sigma T_s C(v'' - v')}{r y_a} \tag{6}$$

and the derivative T_a with respect to y_a, thus determining the nature of the variation in the vapor temperature within the bubble as a function of the apex coordinate, i.e., of the bubble dimension

$$\frac{dT_a}{dy_a} = -\frac{2\sigma C T_s (v'' - v')}{r y_a^2}. \tag{7}$$

Now, requiring equality between dT/dy and the dT_a/dy_a from the bubble crises condition (7), in accordance with the law governing change in the temperature profile in the boundary layer we find a possibility of associating the bubble crises condition with the heat flow through the wall to the moving liquid. All subsequent considerations will be based on the dimensionless universal Karman coordinates.

Let us introduce the notation

$$y^+ = \frac{\sqrt{\frac{\tau_0}{\rho_0}}}{\frac{\mu_0}{\rho_0}} y,$$

$$u^+ = \frac{u}{\sqrt{\frac{\tau_0}{\rho_0}}}, \tag{8}$$

$$t^+ = \frac{c_{p0}(T_0 - T_s)\rho_0 \sqrt{\frac{\tau_0}{\rho_0}}}{q_0}.$$

With consideration of (8) we find that (7) assumes the form

$$\frac{dt_a^+}{dy_a^+} = \frac{2\sigma C T_s c_{p0} \tau_0}{r \mu_0 q_0} \left(\frac{v''}{v'} - 1\right) \frac{1}{y_a^{+^2}}. \tag{9}$$

The quantity τ_0 in (9) can be represented in terms of the pressure gradient (dP/dx) for a liquid flow by a complete cross section or in terms of the thickness of the liquid film freely running off a vertical surface.

In the first case,

$$\tau_0 = \frac{R}{2}\left(\frac{dP}{dx}\right), \tag{10}$$

where R is the tube radius.

In the second case,

$$\tau_0 = (\mu^2 \rho g^2)^{1/3} \delta^{+2/3}.$$

The magnitude of the pressure gradient dP/dx in (10) is determined on the saturation line for a one-phase liquid as a function of the flow regime. In the region of turbulent regimes the pressure gradient can be calculated from the formula

$$\frac{dP}{dx} = \frac{\zeta}{D}\frac{\bar{u}^2\rho}{2}. \tag{11}$$

Substituting (11) into the expression for the dynamic velocity $u^* = \sqrt{\tau_0/\rho_0}$, we obtain

$$u^* = \bar{u}\sqrt{\frac{\zeta}{8}}. \tag{12}$$

For turbulent motion in a circular tube, according to the recommendations of [3], the resistance factor ζ can be calculated from the formula for smooth tubes:

$$\zeta = \left[\frac{0.55}{\lg\frac{\mathrm{Re}}{8}}\right]^2, \tag{13}$$

and with consideration of the above Eq. (12) assumes the form

$$u^* = \frac{0.195}{\lg\frac{\mathrm{Re}}{8}}\bar{u}.$$

The quantity δ^+ in (10a) is the dimensionless thickness of the run-off film, calculated by the Portalski method [4].

The next step in the solution of the problem calls for knowledge of the velocity profile in the liquid boundary layer. For this we will use the universal Karman–Nikuradse velocity profile in a three-layer model of the flow, assuming the tangential stresses in the boundary layer to be constant. The laminar and buffer transition regions may be of practical interest in view of the smallness of the critical bubble dimensions, which in the coordinates adopted here will be determined by the range of variation in y^+ in the limits $0 < y^+ \leq 30$.

In the laminar region $0 < y^+ \leq 5$

$$t^+ = \frac{(T_0 - T_s)\, c_{p0}\rho_0 \,\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}}{q_0}\, y^+, \tag{14}$$

$$\frac{dt^+}{dy^+} = \frac{(T_0 - T_s)\, c_{p0}\rho_0 \,\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}}{q_0}. \tag{15}$$

In the buffer region

$$t^+ = \frac{(T_0 - T_s)\left(5\,\mathrm{Pr} - 5\ln\left(1 - \mathrm{Pr} + \frac{\mathrm{Pr}}{5}y^+\right)\right) c_{p0}\rho_0\sqrt{\frac{\tau_0}{\rho_0}}}{q_0}, \tag{16}$$

$$\frac{dt^+}{dy^+} = \frac{(T_0 - T_s)\, c_{p0}\rho_0 \,\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}}{q_0\left(1 - \mathrm{Pr} + \frac{\mathrm{Pr}}{5}y^+\right)}. \tag{17}$$

Equating the values of dt^+/dy^+ from (9) and (15) or (17), respectively, we will find for the laminar region that

$$y_a^+ = \left(\frac{2\sigma C\sqrt{\frac{\tau_0}{\rho_0}}\left(\frac{v''}{v'} - 1\right)}{\left(\frac{T_0}{T_s} - 1\right) r\mu_0 \mathrm{Pr}}\right)^{1/2}. \tag{18}$$

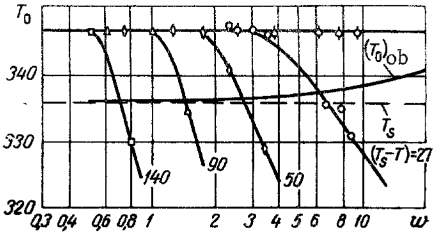

Fig. 2. Comparison of the boundary of the onset of boiling $(T_0)_{ob}$ as a function of (24) with the data from [5] for a pressure of P = 14.0 MN/m^2 (w, m/sec).

This result is valid if the value of $y_a^+ \leq 5$. When $y_a^+ > 5$, i.e., for the buffer region, the equation is brought to the form

$$(y_a^+)^2 - \frac{2\sigma C\sqrt{\frac{\tau_0}{\rho_0}}\left(\frac{v''}{v'}-1\right)}{5\left(\frac{T_0}{T_s}-1\right)r\mu_0}\, y_a^+ - \frac{2\sigma C\sqrt{\frac{\tau_0}{\rho_0}}\left(\frac{v''}{v'}-1\right)(1-\mathrm{Pr})}{r\mu_0\mathrm{Pr}\left(\frac{T_0}{T_s}-1\right)} = 0. \tag{19}$$

The solution satisfying the physical sense of the problem ($y_a^+ > 0$) has the form

$$y_a^+ = \frac{\sigma C\left(\frac{v''}{v'}-1\right)\sqrt{\frac{\tau_0}{\rho_0}}}{5r\mu_0\left(\frac{T_0}{T_s}-1\right)}\left(1+\sqrt{1-\frac{50(\mathrm{Pr}-1)r\mu_0\left(\frac{T_0}{T_s}-1\right)}{\sigma C\left(\frac{v''}{v'}-1\right)\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}}}\right). \tag{20}$$

Let us determine which external conditions lead to the critical state for vapor bubbles with a dimension of y_a^+ from (18) or (20), depending on the absolute value.

Substituting the value of y_a^+ from (18) into (6) we find the temperature inside the bubble:

$$t_a^+ = \frac{c_{p0}\rho_0\sqrt{\frac{\tau_0}{\rho_0}}}{q_0}(T_0-T_s)\left\{1-\left[\frac{2\sigma C\left(\frac{v''}{v'}-1\right)\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}}{\left(\frac{T_0}{T_s}-1\right)r\mu_0}\right]^{\frac{1}{2}}\right\}. \tag{21}$$

We will use (14) when $y^+ = y_a^+$ according to (19) to eliminate t_a^+ from (22). This yields

$$\left[\frac{2\sigma C\,\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}\left(\frac{v''}{v'}-1\right)}{r\mu_0\left(\frac{T_0}{T_s}-1\right)}\right]^{1/2} = \frac{1}{2}. \tag{22}$$

With simple transformations, from this we determine the value of the temperature head, corresponding to the onset of nucleate boiling in the forced flow of a liquid:

$$(T_0-T_s) = \frac{8T_s\sigma C\,\mathrm{Pr}\sqrt{\frac{\tau_0}{\rho_0}}\left(\frac{v''}{v'}-1\right)}{r\mu_0}. \tag{23}$$

We can present (23) in dimensionless form, i.e.,

$$\left(\frac{T_0}{T_s}-1\right) = 8C\left(\frac{v''}{v'}-1\right)\mathrm{Re}^*\ \mathrm{Pr}, \tag{24}$$

where

$$\mathrm{Re}^* = \frac{\frac{\sigma}{r\rho_0}\sqrt{\frac{\tau_0}{\rho_0}}\,\rho_0}{\mu_0}.$$

The complex $\sigma/r\rho_0$ in the dynamic Reynolds number is expressed in units of meters and is taken as a characteristic dimension.

Returning to (18) for the dimensionless coordinate of the apex of the critical bubble, we eliminate the quantity $(T_0 - T_s)$ from that equation on the basis of (22). This yields the condition

$$y_a^+ = \frac{1}{2\mathrm{Pr}} \leqslant 5, \tag{25}$$

which demonstrates that the critical bubble is entirely within the limits of the laminar boundary layer. Analyzing the various heat carriers from this standpoint, we can see that most of them satisfy this condition. An exception are the liquid metals for which the Pr numbers on saturation are of the order of 10^{-2}. For liquid-metal heat carriers it is therefore of interest to examine the case in which the apex of the vapor formation is in the buffer region.

Substituting the value of y_a^+ from (20) into (16) and (6), and eliminating the value of t_a^+, after some simple transformations we obtain

$$1 - \mathrm{Pr} + \frac{\mathrm{Pr}}{25} \frac{C\left(\frac{v''}{v'} - 1\right)}{\left(\frac{T_0}{T_s} - 1\right)} \mathrm{Re}^* \left(1 + \sqrt{1 + \frac{50\left(\frac{1}{\mathrm{Pr}} - 1\right)\left(\frac{T_0}{T_s} - 1\right)}{\left(\frac{v''}{v'} - 1\right) C\,\mathrm{Re}^*}}\right)$$
$$= \exp\left[5\,\mathrm{Pr} - 1 + \frac{10}{1 + \sqrt{1 + \frac{50\left(\frac{1}{\mathrm{Pr}} - 1\right)\left(\frac{T_0}{T_s} - 1\right)}{\left(\frac{v''}{v'} - 1\right) C\,\mathrm{Re}^*}}}\right]. \tag{26}$$

The solution of the transcendental equation (26) for $(T_0/T_s - 1)$ determines the minimum temperature head at which nucleate boiling begins under conditions of forced liquid flow ($\mathrm{Pr} < 0.1$).

The resulting relationship (24) for the minimum temperature difference corresponding to the onset of nucleate boiling in a liquid flow was compared with experimental data [5, 7] and the empirical relationships of [6]. The results are shown in Figs. 1 and 2.

It follows from Fig. 2 that in a region of high pressures the curve calculated from (24) corresponds to the onset of deviation on the part of the wall temperature from the convection heat-transfer relationships for various degrees of liquid subcooling to saturation.

With an error of less than 20%, the experimental data of [7] were grouped near the theoretical curve corresponding to pressure of $1.5 \cdot 10^5$ N/m^2 and this curve is shown in Fig. 1. The same figure also shows the curves calculated from the empirical relationships of [6] for a subcooling of 20°. In the region of high pressures and low velocities for the liquid we find satisfactory agreement for the relationships under consideration. It is characteristic that the divergence of the result is not systematic in nature and may be partially explained by the fact that in the processing of the experimental data the authors of [6] employed the total temperature head between the wall and the core of the flow, thus distorting the effect of pressure and velocity.

NOTATION

P is the pressure in the system;
σ is the surface tension coefficient;
R is the radius;
t, T are temperatures;
v is the specific volume of the medium;
r is the heat of vapor formation;
y is the coordinate in the direction normal to the surface;
D is the diameter;
θ is the boundary wetting angle;
u is the instantaneous value of the liquid velocity;
ρ is the liquid density;
τ is the tangential stress;
c_p is the heat capacity;
q is the heat flux density;
μ is the dynamic viscosity;

x is the coordinate along the liquid flow;
g is the acceleration of free fall;
δ is the thickness of the liquid film running off freely;
Δ is an increment in magnitude;
ζ is the frictional resistance factor.

Symbols

" denotes quantities pertaining to the vapor phase;
' denotes quantities pertaining to the liquid phase;
a shows the value of the quantity at the apex of a bubble of critical radius;
s shows the value on saturation;
cr denotes quantities characterizing the critical state;
0 denotes values at the wall temperature;
+ denotes quantities expressed in dimensionless Karman coordinates;
* denotes dynamic velocity and quantities expressed in terms of that velocity;
ob denotes quantities characterizing the onset of boiling.

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